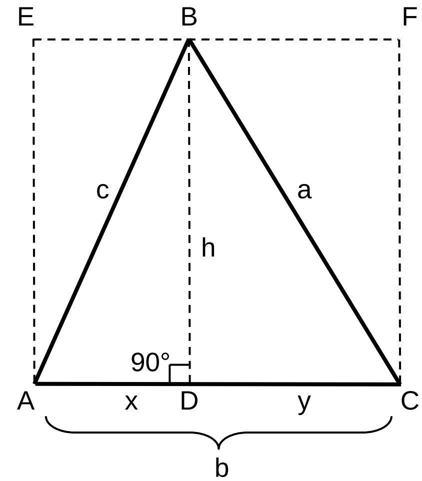
Area ABD = Area ABE = $\frac{x \cdot h}{2}$

Area CBD = Area CBF = $\frac{y \cdot h}{2}$



Therefore…

Area of Triangle ABC…

$$Area = \frac{b \cdot h}{2}$$

$$h = a \cdot \sin C$$

Therefore…

Area of Triangle ABC…

$$Area = \frac{a \cdot b \cdot \sin C}{2}$$

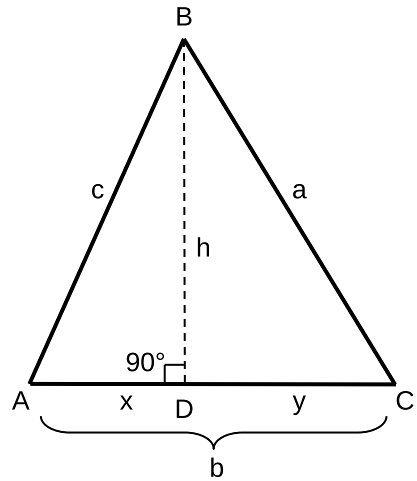
$$b = \frac{a \cdot \sin B}{\sin A}$$

Therefore…

Area of Triangle ABC…

$$Area = \frac{a^2 \cdot \sin B \cdot \sin C}{2 \cdot \sin A}$$

HERON'S FORMULA DERIVATION



Equation 1 $$Area = \frac{b \cdot h}{2}$$

From triangle ABD… $$x^2 + h^2 = c^2$$

Equation 2 $$x^2 = c^2 - h^2$$

Equation 3 $$x = \sqrt{c^2 - h^2}$$

From triangle CBD… $$(b-x)^2 + h^2 = a^2$$

$$(b-x)^2 = a^2 - h^2$$

Equation 4 $$b^2 - 2bx + x^2 = a^2 - h^2$$

Substitute x and x^2 from Equations 3 and 2 into Equation 4 …

$$b^2 - 2b\sqrt{c^2 - h^2} + (c^2 - h^2) = a^2 - h^2$$

$$b^2 + c^2 - a^2 = 2b\sqrt{c^2 - h^2}$$

Square both sides … $$(b^2 + c^2 - a^2)^2 = 4b^2(c^2 - h^2)$$

$$\frac{(b^2+c^2-a^2)^2}{4b^2} = c^2 - h^2$$

$$h^2 = c^2 - \frac{(b^2+c^2-a^2)^2}{4b^2}$$

$$h^2 = \frac{4b^2c^2-(b^2+c^2-a^2)^2}{4b^2}$$

$$h^2 = \frac{(2bc)^2-(b^2+c^2-a^2)^2}{4b^2}$$

$$h^2 = \frac{[2bc+(b^2+c^2-a^2)]\cdot[2bc-(b^2+c^2-a^2)]}{4b^2}$$

$$h^2 = \frac{[2bc+b^2+c^2-a^2]\cdot[2bc-b^2-c^2+a^2]}{4b^2}$$

$$h^2 = \frac{[(b^2+2bc+c^2)-a^2]\cdot[a^2-(b^2-2bc+c^2)]}{4b^2}$$

$$h^2 = \frac{\left[(b+c)^2-a^2\right]\cdot\left[a^2-(b-c)^2\right]}{4b^2}$$

$$h^2 = \frac{[(b+c)+a]\cdot[(b+c)-a]\cdot[a+(b-c)]\cdot[a-(b-c)]}{4b^2}$$

$$h^2 = \frac{(b+c+a)(b+c-a)(a+b-c)(a-b+c)}{4b^2}$$

$$h^2 = \frac{(a+b+c)(b+c-a)(a+c-b)(a+b-c)}{4b^2}$$

$$h^2 = \frac{(a+b+c)(a+b+c-2a)(a+b+c-2b)(a+b+c-2c)}{4b^2}$$

Since $P = a + b + c$ …

$$h^2 = \frac{P(P-2a)(P-2b)(P-2c)}{4b^2}$$

<u>Equation 5</u>

$$h = \frac{\sqrt{P(P-2a)(P-2b)(P-2c)}}{2b}$$

Substitute h from Equation 5 into Equation 1 …

$$Area = \frac{1}{2}b\,\frac{\sqrt{P(P-2a)(P-2b)(P-2c)}}{2b}$$

$$Area = \frac{1}{4}\sqrt{P(P-2a)(P-2b)(P-2c)}$$

$$Area = \sqrt{\frac{1}{16}P(P-2a)(P-2b)(P-2c)}$$

$$Area = \sqrt{\left(\frac{P}{2}\right)\left(\frac{P-2a}{2}\right)\left(\frac{P-2b}{2}\right)\left(\frac{P-2c}{2}\right)}$$

$$Area = \sqrt{\frac{P}{2}\left(\frac{P}{2}-a\right)\left(\frac{P}{2}-b\right)\left(\frac{P}{2}-c\right)}$$

Thus...

Given the three sides of a triangle (a, b, and c) …

The area of the triangle is:

$$Area = \sqrt{s(s-a)(s-b)(s-c)}$$

Where the semi perimeter is:

$$s = P/2 = (a+b+c)/2$$